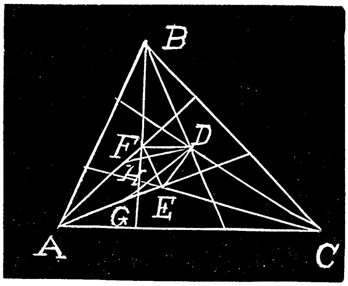
370. Proposed by R. C. ARCHIBALD, Paris, France.

The trisectors of the angles of any triangle ABC are, in order, AF, AE, CE, CD, BD, BF. Show synthetically that D, E, F are the vertices of an equilateral triangle.

Solution by A. H. HOLMES, Bruntwick, Me.

Let ABC be the triangle with AE, AF trisectors of $\angle BAC$; BF, BD trisectors of $\angle ABC$, and CE, CD trisectors of $\angle ACB$. Draw DE, DF, and EF. Then DEF is an equilateral triangle. From D draw DG parallel to AF and cutting AE in G. Take, on AF, $AH=DG$. Then $\angle GDH=\angle EAF=\frac{1}{3}\angle BAC$.



Suppose the triangle GED to be moved so that GE is colinear with AF and the point E is at the point F. Then since DG is parallel to AF and $\angle EAF=\angle FAB$, DG will be parallel to AB.

$\therefore \triangle DEG$ and $\triangle AFB$ are similar, and $\angle EDG=\angle ABF=\frac{1}{3}\angle ABC$. Similarly, it may be shown that $\angle FDH$ is equal to $\frac{1}{3}\angle ACB$.

$\therefore \angle EDF=\frac{1}{3}(\angle ABC+\angle ACD+\angle BAC)=60°$. In the same way, $\angle DEF$ is shown to be equal to $60°$.

$\therefore \triangle DEF$ is an equilateral triangle.

371. Proposed by W. S. HUGHES, Student, Williams College.

A right circular cone is cut by two parallel planes, one passing through the vertex, and each cutting both nappes. Are the straight lines which constitute the first section parallel to the asymptotes of the hyperbola forming the other section?

Solution by FRANK LOXLEY GRIFFIN, Ph. D., Assistant Professor of Mathematics, Williams College.

As the X-axis take the axis of the cone, and as the Y-axis the intersection of the first cutting plane with the plane through the vertex perpendicular to the X-axis. Then the equation of the cone is $y^2+z^2=m^2x^2$, where m denotes the tangent of one-half the vertex angle of the cone. In the plane XOZ rotate the axes OX and OZ through an angle α, such that OX shall be in the first cutting plane. Then the cone and the cutting planes are given respectively by

(1) $$y^2+(x'\sin\alpha+z'\cos\alpha)^2=m^2(x'\cos\alpha-z'\sin\alpha),$$

(2), (3) $$z'=0, \qquad z'=d,$$

where d is the distance between the cutting planes. Now the straight lines of the first section are given by (2) and the equation obtained by making $z'=0$ in (1), say $B^2x'^2-y^2=0$. And the hyperbola is given by (3) and an